The Role of the Fibonacci Sequence in the Collatz 3n + 1 Function

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Abstract

The number theoretic function $T(n) = \frac{n}{2}$ if n is even, $T(n) = \frac{3n+1}{2}$ if n is odd, generates for each starting number $s \in \mathbb{N}$ a Collatz sequence $C(s) = \left(T^k(s)_{k=0}^{\infty}\right), T^0(s) = s, T^k(s) = T\left(T^{k-1}(s)\right)$. Every $C(s)$ consists of same structured subsequences $C^h(s) = \left(T^k(s)_{k=0}^{h}\right)$ for $s \equiv 9\ (mod\ 12)$ or $C^t(s) = \left(T^k(s)_{k=0}^{t}\right)$ for $s \equiv 3, 7\ (mod\ 12)$. For starting numbers of specific residue classes the subsequences have same length h, t. It is shown that for each $h, t \geq 2$ the number of all admissible residue classes is given exactly by the Fibonacci sequence.

1. Introduction

The Collatz $3n + 1$ function is defined as a function $T : \mathbb{N} \to \mathbb{N}$ on the set of positive integers by

$$T(n) := \begin{cases} T_0 := \dfrac{n}{2} & \text{if } n \text{ is even,} \\ \\ T_1 := \dfrac{3n+1}{2} & \text{if } n \text{ is odd.} \end{cases}$$

Let $T^0(s) = s$ and $T^k(s) = T\left(T^{k-1}(s)\right)$ for $k \in \mathbb{N}$. Then the Collatz sequence for $s \in \mathbb{N}$ is $C(s) = \left(T^k(s) \mid k = 0, 1, 2, 3, \dots\right)$.

For example, the starting number $s = 11$ generates the Collatz sequence

$$C(11) = (11, 17, 26, 13, 20, 10, 5, 8, 4, 2, 1, 2, 1, 2, 1, \dots).$$

A Collatz sequence can only assume two possible forms. Either it falls into a cycle or it grows to infinity. The unproved conjecture to this problem is that each Collatz sequence enters the cycle $(2, 1)$.

2. Finite subsequences

Let $C^h(s) = \big(T^k(s) \mid k = 0, \ldots, h\big)$ with $h \geq 2$ be a finite subsequence of $C(s)$ for each $s \equiv 9 \ (mod\ 12)$, in which for the least h is $T^h(s) = 1$ or $T^h(s) \equiv 3 \ (mod\ 4)$. Then the first subsequences $C^h(s)$ are

$$
\begin{aligned}
C^2(9) &= (9, 14, 7),\\
C^6(21) &= (21, 32, 16, 8, 4, 2, 1),\\
C^4(33) &= (33, 50, 25, 38, 19,)\\
C^{12}(45) &= (45, 68, 34, 17, 26, 13, 20, 10, 5, 8, 4, 2, 1),\\
C^2(57) &= (57, 86, 43),\\
C^{11}(69) &= (69, 104, 52, 26, 13, 20, 10, 5, 8, 4, 2, 1),\\
C^5(81) &= (81, 122, 61, 92, 46, 23),\\
C^2(93) &= (93, 140, 70, 35),\\
&\textit{and so forth.}
\end{aligned}
$$

Let $C^t(s) = \big(T^k(s) \mid k = 0, \ldots, t\big)$ with $t \geq 2$ be a finite subsequence of $C(s)$ for each $s \equiv 3, 7 \ (mod\ 12)$, in which for the least t is $T^t(s) = 1$ or $T^t(s) \equiv 6 \ (mod\ 8)$. Then the first subsequences $C^t(s)$ are

$$
\begin{aligned}
C^5(3) &= (3, 5, 8, 4, 2, 1),\\
C^{11}(7) &= (7, 11, 17, 26, 13, 20, 10, 5, 8, 4, 2, 1),\\
C^{12}(15) &= (15, 23, 35, 53, 80, 40, 20, 10, 5, 8, 4, 2, 1),\\
C^3(19) &= (19, 29, 44, 22),\\
C^2(27) &= (27, 41, 62),\\
C^7(31) &= (31, 47, 71, 107, 161, 242, 121, 182),\\
C^3(39) &= (39, 59, 89, 134),\\
C^8(43) &= (43, 65, 98, 49, 74, 37, 56, 28, 14),\\
&\textit{and so forth.}
\end{aligned}
$$

Appendix 7.1 and 7.2 show lists of the first subsequences $C^h(s)$ up to $s = 2085$ and $C^t(s)$ up to $s = 1027$.

2.1 How the subsequences works

For a better understanding of how a Collatz sequence consists out of their subsequences let us take a look on the sequence with the starting number $s = 27$.

$C(27) = (27, 41, 62, 31, 47, 71, 107, 161, 242, 121, 182, 91, 137, 206, 103, 155, 233, 350, 175, 263, 395, 593, 890, 445, 668, 334, 167, 251, 377, 566, 283, 425, 638, 319, 479, 719, 1079, 1619, 2429, 3644, 1822, 911, 1367, 2051, 3077, 4616, 2308, 1154, 577, 866, 433, 650, 325, 488, 244, 122, 61, 92, 46, 23, 35, 53, 80, 40, 20, 10, 5, 8, 4, 2, 1)$.

This sequence consists of 59 terms, which can be subdivided into 10 subsequences $C^t(s)$.

(27, 41, 62),
(31, 47, 71, 107, 161, 242, 121, 182),
(91, 137, 206),
(103, 155, 233, 350),
(175, 263, 395, 593, 890, 445, 668, 334),
(111, 167, 251, 377, 566),
(283, 425, 638),
(319, 479, 719, 1079, 1619, 2429, 3644, 1822),
(607, 911, 1367, 2051, 3077, 4616, 2308, 1154, 577, 866, 433, 650, 325, 488, 244, 122, 61, 92, 46),
(15, 23, 35, 53, 80, 40, 20, 10, 5, 8, 4, 2, 1).

Note: Every Collatz sequence $C(s)$ consists only of these two forms of same structured finite subsequences $C^h(s)$ and $C^t(s)$. For details see Winkler[7].

3. Finite subsequences of same length

It can be observed that for starting numbers of specific residue classes $(mod\ 12 \cdot 2^h)$ or $(mod\ 12 \cdot 2^{t+1})$ the subsequences have the same length h or t.

For the $C^h(s)$ it is

$h = 2$ if $s \equiv 9\ (mod\ 48)$,
$h = 3$ if $s \equiv 93\ (mod\ 96)$,
$h = 4$ if $s \equiv 33, 165\ (mod\ 192)$,
$h = 5$ if $s \equiv 81, 117, 237\ (mod\ 384)$,
$h = 6$ if $s \equiv 129, 333, 405, 561, 645\ (mod\ 768)$,
$h = 7$ if $s \equiv 429, 657, 837, 981, 1293, 1461, 1473, 1521\ (mod\ 1536)$,
$h = 8$ if $s \equiv 177, 309, 513, 597, 1089, 1221, 1557, 1581, \ldots\ (mod\ 3072)$,
and so forth.

For the $C^t(s)$ it is

$t = 2$ if $s \equiv 27, 91\ (mod\ 96)$,
$t = 3$ if $s \equiv 19, 39, 103, 147\ (mod\ 192)$,
$t = 4$ if $s \equiv 55, 67, 111, 183, 195, 235, 363, 367\ (mod\ 384)$,

$t = 5$ if $s \equiv 139, 159, 163, 207, 243, 327, 415, \ldots \ (mod\ 768)$,
$t = 6$ if $s \equiv 51, 99, 259, 279, 427, 447, 559, 655, \ldots \ (mod\ 1536)$,
$t = 7$ if $s \equiv 31, 135, 175, 291, 319, 331, 375, 627, \ldots \ (mod\ 3072)$,
$t = 8$ if $s \equiv 43, 63, 199, 223, 271, 351, 355, 435, 519, \ldots \ (mod\ 6144)$,
and so forth.

Appendix 7.3 and 7.4 show lists of all admissible residue classes for the $C^h(s)$ and the $C^t(s)$ up to $h = 16$ and $t = 14$.

4. The role of the Fibonacci sequence

When counting the residue classes of same length subsequences, it is noticeable that for each $h, t \geq 2$ the number of residue classes for the $C^h(s)$ is given exactly by the Fibonacci sequence (OEIS A000045) and for the $C^t(s)$ exactly by a sequence based on the Fibonacci sequence (OEIS A019274).

Let $A(h)$ be the number of residue classes $(mod\ 12 \cdot 2^h)$, then for each $h \in \mathbb{N}$, $h \geq 2$ it is

$$A(h) = \frac{1}{\sqrt{5}} \left[\left(\frac{1+\sqrt{5}}{2} \right)^{h-1} - \left(\frac{1-\sqrt{5}}{2} \right)^{h-1} \right].$$

Let $A(t)$ be the number of residue classes $(mod\ 12 \cdot 2^{t+1})$, then for each $t \in \mathbb{N}$, $t \geq 2$ it is

$$A(t) = \frac{2}{\sqrt{5}} \left[\left(\frac{1+\sqrt{5}}{2} \right)^{t+1} - \left(\frac{1-\sqrt{5}}{2} \right)^{t+1} \right] - 2.$$

Chapter 3. and 4. instantly raises a question to a connection to the number of residue classes for Collatz sequences with a finite stopping time.

5. Stopping time

Collatz's conjecture is equivalent to the conjecture that for each $s \in \mathbb{N}, s > 1$, there exists $k \in \mathbb{N}$ such that $T^k(s) < s$. The least $k \in \mathbb{N}$ such that $T^k(s) < s$ is called the stopping time of s, which we will denote by $\sigma(s)$. It is not hard to verify that

$\sigma(s) = 1$ if $s \equiv 0\ (mod\ 2)$,
$\sigma(s) = 2$ if $s \equiv 1\ (mod\ 4)$,
$\sigma(s) = 4$ if $s \equiv 3\ (mod\ 16)$,
$\sigma(s) = 5$ if $s \equiv 11, 23\ (mod\ 32)$,
$\sigma(s) = 7$ if $s \equiv 7, 15, 59\ (mod\ 128)$,
$\sigma(s) = 8$ if $s \equiv 39, 79, 95, 123, 175, 199, 219\ (mod\ 256)$,
$\sigma(s) = 10$ if $s \equiv 287, 347, 367, 423, 507, 575, 583, \ldots\ (mod\ 1024)$,
and so forth.

Generally applies for all $n \in \mathbb{N}$, $n \geq 0$

$$\sigma(s) = \lfloor 1 + n \cdot \log_2 3 \rfloor \quad \text{if} \quad s \equiv s_{n_1}, s_{n_2}, \ldots, s_{n_z}\ (mod\ 2^{\sigma(s)}).$$

Everett[3] proves that almost all $k \in \mathbb{N}$ have finite stopping time, and Terras[4] gives a probability distribution function for stopping times.

The possible stopping times $\sigma(s)$ are listed at OEIS A020914. The associated residue classes $(mod\ 2^{\sigma(s)})$ are listed at OEIS A177789. The number of residue classes $z(n)$ for $n \geq 1$ are listed at OEIS A100982.

Appendix 5.5 shows a list of the first residue classes $(mod\ 2^{\sigma(s)})$ up to $\sigma(s) = 16$.

6. Conclusion and further questions

The shown connection to the Fibonacci sequence is just a heuristic observation. But a mathematical proof of it could be helpful for a proof of the Collatz conjecture, because it extracts a chaotic element out of the distribution of the subsequences. For details see Winkler[7].

This raises the question whether there exists also a formula based on the Fibonacci sequence to generate the numbers of the residue classes for the stopping times (OEIS A100982). The existence of an iterative algorithm based on simple integer arithmetic (without using of the Fibonacci sequence) who generates these numbers is shown by Winkler[5].

If it would be possible to determine the exact residue classes of stopping times (OEIS A177789) by using the Fibonacci sequence, a proof of the Collatz conjecture would be within reach.

7. Appendix

7.1 The first subsequences $C^h(s)$

9, 14, 7
21, 32, 16, 8, 4, 2, 1
33, 50, 25, 38, 19
45, 68, 34, 17, 26, 13, 20, 10, 5, 8, 4, 2, 1
57, 86, 43
69, 104, 52, 26, 13, 20, 10, 5, 8, 4, 2, 1
81, 122, 61, 92, 46, 23
93, 140, 70, 35
105, 158, 79
117, 176, 88, 44, 22, 11
129, 194, 97, 146, 73, 110, 55
141, 212, 106, 53, 80, 40, 20, 10, 5, 8, 4, 2, 1
153, 230, 115
165, 248, 124, 62, 31
177, 266, 133, 200, 100, 50, 25, 38, 19
189, 284, 142, 71
201, 302, 151
213, 320, 160, 80, 40, 20, 10, 5, 8, 4, 2, 1
225, 338, 169, 254, 127
237, 356, 178, 89, 134, 67
249, 374, 187
261, 392, 196, 98, 49, 74, 37, 56, 28, 14, 7
273, 410, 205, 308, 154, 77, 116, 58, 29, 44, 22, 11
285, 428, 214, 107
297, 446, 223
309, 464, 232, 116, 58, 29, 44, 22, 11
321, 482, 241, 362, 181, 272, 136, 68, 34, 17, 26, 13, 20, 10, 5, 8, 4, 2, 1
333, 500, 250, 125, 188, 94, 47
345, 518, 259
357, 536, 268, 134, 67
369, 554, 277, 416, 208, 104, 52, 26, 13, 20, 10, 5, 8, 4, 2, 1
381, 572, 286, 143
393, 590, 295
405, 608, 304, 152, 76, 38, 19
417, 626, 313, 470, 235
429, 644, 322, 161, 242, 121, 182, 91
441, 662, 331
453, 680, 340, 170, 85, 128, 64, 32, 16, 8, 4, 2, 1
465, 698, 349, 524, 262, 131
477, 716, 358, 179
489, 734, 367
501, 752, 376, 188, 94, 47
513, 770, 385, 578, 289, 434, 217, 326, 163
525, 788, 394, 197, 296, 148, 74, 37, 56, 28, 14, 7
537, 806, 403
549, 824, 412, 206, 103
561, 842, 421, 632, 316, 158, 79
573, 860, 430, 215
585, 878, 439
597, 896, 448, 224, 112, 56, 28, 14, 7
609, 914, 457, 686, 343
621, 932, 466, 233, 350, 175
633, 950, 475
645, 968, 484, 242, 121, 182, 91
657, 986, 493, 740, 370, 185, 278, 139
669, 1004, 502, 251
681, 1022, 511
693, 1040, 520, 260, 130, 65, 98, 49, 74, 37, 56, 28, 14, 7
705, 1058, 529, 794, 397, 596, 298, 149, 224, 112, 56, 28, 14, 7
717, 1076, 538, 269, 404, 202, 101, 152, 76, 38, 19
729, 1094, 547
741, 1112, 556, 278, 139
753, 1130, 565, 848, 424, 212, 106, 53, 80, 40, 20, 10, 5, 8, 4, 2, 1
765, 1148, 574, 287
777, 1166, 583
789, 1184, 592, 296, 148, 74, 37, 56, 28, 14, 7
801, 1202, 601, 902, 451
813, 1220, 610, 305, 458, 229, 344, 172, 86, 43
825, 1238, 619
837, 1256, 628, 314, 157, 236, 118, 59
849, 1274, 637, 956, 478, 239
861, 1292, 646, 323
873, 1310, 655
885, 1328, 664, 332, 166, 83
897, 1346, 673, 1010, 505, 758, 379
909, 1364, 682, 341, 512, 256, 128, 64, 32, 16, 8, 4, 2, 1
921, 1382, 691
933, 1400, 700, 350, 175
945, 1418, 709, 1064, 532, 266, 133, 200, 100, 50, 25, 38, 19
957, 1436, 718, 359
969, 1454, 727
981, 1472, 736, 368, 184, 92, 46, 23
993, 1490, 745, 1118, 559
1005, 1508, 754, 377, 566, 283
1017, 1526, 763

1029, 1544, 772, 386, 193, 290, 145, 218, 109, 164, 82, 41, 62, 31
1041, 1562, 781, 1172, 586, 293, 440, 220, 110, 55
1053, 1580, 790, 395
1065, 1598, 799
1077, 1616, 808, 404, 202, 101, 152, 76, 38, 19
1089, 1634, 817, 1226, 613, 920, 460, 230, 115
1101, 1652, 826, 413, 620, 310, 155
1113, 1670, 835
1125, 1688, 844, 422, 211
1137, 1706, 853, 1280, 640, 320, 160, 80, 40, 20, 10, 5, 8, 4, 2, 1
1149, 1724, 862, 431
1161, 1742, 871
1173, 1760, 880, 440, 220, 110, 55
1185, 1778, 889, 1334, 667
1197, 1796, 898, 449, 674, 337, 506, 253, 380, 190, 95
1209, 1814, 907
1221, 1832, 916, 458, 229, 344, 172, 86, 43
1233, 1850, 925, 1388, 694, 347
1245, 1868, 934, 467
1257, 1886, 943
1269, 1904, 952, 476, 238, 119
1281, 1922, 961, 1442, 721, 1082, 541, 812, 406, 203
1293, 1940, 970, 485, 728, 364, 182, 91
1305, 1958, 979
1317, 1976, 988, 494, 247
1329, 1994, 997, 1496, 748, 374, 187
1341, 2012, 1006, 503
1353, 2030, 1015
1365, 2048, 1024, 512, 256, 128, 64, 32, 16, 8, 4, 2, 1
1377, 2066, 1033, 1550, 775
1389, 2084, 1042, 521, 782, 391
1401, 2102, 1051
1413, 2120, 1060, 530, 265, 398, 199
1425, 2138, 1069, 1604, 802, 401, 602, 301, 452, 226, 113, 170, 85, 128, 64, 32, 16, 8, 4, 2, 1
1437, 2156, 1078, 539
1449, 2174, 1087
1461, 2192, 1096, 548, 274, 137, 206, 103
1473, 2210, 1105, 1658, 829, 1244, 622, 311
1485, 2228, 1114, 557, 836, 418, 209, 314, 157, 236, 118, 59
1497, 2246, 1123
1509, 2264, 1132, 566, 283
1521, 2282, 1141, 1712, 856, 428, 214, 107
1533, 2300, 1150, 575
1545, 2318, 1159
1557, 2336, 1168, 584, 292, 146, 73, 110, 55
1569, 2354, 1177, 1766, 883
1581, 2372, 1186, 593, 890, 445, 668, 334, 167
1593, 2390, 1195
1605, 2408, 1204, 602, 301, 452, 226, 113, 170, 85, 128, 64, 32, 16, 8, 4, 2, 1
1617, 2426, 1213, 1820, 910, 455
1629, 2444, 1222, 611
1641, 2462, 1231
1653, 2480, 1240, 620, 310, 155
1665, 2498, 1249, 1874, 937, 1406, 703
1677, 2516, 1258, 629, 944, 472, 236, 118, 59
1689, 2534, 1267
1701, 2552, 1276, 638, 319
1713, 2570, 1285, 1928, 964, 482, 241, 362, 181, 272, 136, 68, 34, 17, 26, 13, 20, 10, 5, 8 4, 2, 1
1725, 2588, 1294, 647
1737, 2606, 1303
1749, 2624, 1312, 656, 328, 164, 82, 41, 62, 31
1761, 2642, 1321, 1982, 991
1773, 2660, 1330, 665, 998, 499
1785, 2678, 1339
1797, 2696, 1348, 674, 337, 506, 253, 380, 190, 95
1809, 2714, 1357, 2036, 1018, 509, 764, 382, 19
1821, 2732, 1366, 683
1833, 2750, 1375
1845, 2768, 1384, 692, 346, 173, 260, 130, 65, 98, 49, 74, 37, 56, 28, 14, 7
1857, 2786, 1393, 2090, 1045, 1568, 784, 392, 196, 98, 49, 74, 37, 56, 28, 14, 7
1869, 2804, 1402, 701, 1052, 526, 263
1881, 2822, 1411
1893, 2840, 1420, 710, 355
1905, 2858, 1429, 2144, 1072, 536, 268, 134, 67
1917, 2876, 1438, 719
1929, 2894, 1447
1941, 2912, 1456, 728, 364, 182, 91
1953, 2930, 1465, 2198, 1099
1965, 2948, 1474, 737, 1106, 553, 830, 415
1977, 2966, 1483
1989, 2984, 1492, 746, 373, 560, 280, 140, 70, 35
2001, 3002, 1501, 2252, 1126, 563
2013, 3020, 1510, 755
2025, 3038, 1519
2037, 3056, 1528, 764, 382, 191
2049, 3074, 1537, 2306, 1153, 1730, 865, 1298, 649, 974, 487
2061, 3092, 1546, 773, 1160, 580, 290, 145, 218, 109, 164, 82, 41, 62, 31
2073, 3110, 1555
2085, 3128, 1564, 782, 391

and so forth.

7.2 The first subsequences $C^t(s)$

3, 5, 8, 4, 2, 1
7, 11, 17, 26, 13, 20, 10, 5, 8, 4, 2, 1
15, 23, 35, 53, 80, 40, 20, 10, 5, 8, 4, 2, 1
19, 29, 44, 22
27, 41, 62
31, 47, 71, 107, 161, 242, 121, 182
39, 59, 89, 134
43, 65, 98, 49, 74, 37, 56, 28, 14
51, 77, 116, 58, 29, 44, 22
55, 83, 125, 188, 94
63, 95, 143, 215, 323, 485, 728, 364, 182
67, 101, 152, 76, 38
75, 113, 170, 85, 128, 64, 32, 16, 8, 4, 2, 1
79, 119, 179, 269, 404, 202, 101, 152, 76, 38
87, 131, 197, 296, 148, 74, 37, 56, 28, 14
91, 137, 206
99, 149, 224, 112, 56, 28, 14
103, 155, 233, 350
111, 167, 251, 377, 566
115, 173, 260, 130, 65, 98, 49, 74, 37, 56, 28, 14
123, 185, 278,
127, 191, 287, 431, 647, 971, 1457, 2186, 1093, 1640, 820, 410, 205, 308, 154, 77, 116, 58, 29, 44, 22
135, 203, 305, 458, 229, 344, 172, 86
139, 209, 314, 157, 236, 118
147, 221, 332, 166
151, 227, 341, 512, 256, 128, 64, 32, 16, 8, 4, 2, 1
159, 239, 359, 539, 809, 1214
163, 245, 368, 184, 92, 46
171, 257, 386, 193, 290, 145, 218, 109, 164, 82, 41, 62
175, 263, 395, 593, 890, 445, 668, 334
183, 275, 413, 620, 310
187, 281, 422
195, 293, 440, 220, 110
199, 299, 449, 674, 337, 506, 253, 380, 190
207, 311, 467, 701, 1052, 526
211, 317, 476, 238
219, 329, 494
223, 335, 503, 755, 1133, 1700, 850, 425, 638
231, 347, 521, 782
235, 353, 530, 265, 398
243, 365, 548, 274, 137, 206
247, 371, 557, 836, 418, 209, 314, 157, 236, 118
255, 383, 575, 863, 1295, 1943, 2915, 4373, 6560, 3280, 1640, 820, 410, 205, 308, 154, 77, 116, 58, 29, 44, 22
259, 389, 584, 292, 146, 73, 110
267, 401, 602, 301, 452, 226, 113, 170, 85, 128, 64, 32, 16, 8, 4, 2, 1
271, 407, 611, 917, 1376, 688, 344, 172, 86
279, 419, 629, 944, 472, 236, 118
283, 425, 638
291, 437, 656, 328, 164, 82, 41, 62
295, 443, 665, 998
303, 455, 683, 1025, 1538, 769, 1154, 577, 866, 433, 650, 325, 488, 244, 122, 61, 92, 46
307, 461, 692, 346, 173, 260, 130, 65, 98, 49, 74, 37, 56, 28, 14
315, 473, 710
319, 479, 719, 1079, 1619, 2429, 3644, 1822
327, 491, 737, 1106, 553, 830
331, 497, 746, 373, 560, 280, 140, 70
339, 509, 764, 382
343, 515, 773, 1160, 580, 290, 145, 218, 109, 164, 82, 41, 62
351, 527, 791, 1187, 1781, 2672, 1336, 668, 334
355, 533, 800, 400, 200, 100, 50, 25, 38
363, 545, 818, 409, 614
367, 551, 827, 1241, 1862
375, 563, 845, 1268, 634, 317, 476, 238
379, 569, 854
387, 581, 872, 436, 218, 109, 164, 82, 41, 62
391, 587, 881, 1322, 661, 992, 496, 248, 124, 62
399, 599, 899, 1349, 2024, 1012, 506, 253, 380, 190
403, 605, 908, 454
411, 617, 926
415, 623, 935, 1403, 2105, 3158
423, 635, 953, 1430
427, 641, 962, 481, 722, 361, 542
435, 653, 980, 490, 245, 368, 184, 92, 46
439, 659, 989, 1484, 742
447, 671, 1007, 1511, 2267, 3401, 5102
451, 677, 1016, 508, 254
459, 689, 1034, 517, 776, 388, 194, 97, 146, 73, 110
463, 695, 1043, 1565, 2348, 1174
471, 707, 1061, 1592, 796, 398
475, 713, 1070
483, 725, 1088, 544, 272, 136, 68, 34, 17, 26, 13, 20, 10, 5, 8, 4, 2, 1
487, 731, 1097, 1646
495, 743, 1115, 1673, 2510
499, 749, 1124, 562, 281, 422
507, 761, 1142
511, 767, 1151, 1727, 2591, 3887, 5831, 8747, 13121, 19682, 9841, 14762, 7381, 11072, 5536, 2768, 1384, 692, 346,
173, 260, 130, 65, 98, 49, 74, 37, 56, 28, 14
519, 779, 1169, 1754, 877, 1316, 658, 329, 494

523, 785, 1178, 589, 884, 442, 221, 332, 166
531, 797, 1196, 598
535, 803, 1205, 1808, 904, 452, 226, 113, 170, 85, 128, 64, 32, 16, 8, 4, 2, 1
543, 815, 1223, 1835, 2753, 4130, 2065, 3098, 1549, 2324, 1162, 581, 872, 436, 218, 109, 164, 82, 41, 62
547, 821, 1232, 616, 308, 154, 77, 116, 58, 29, 44, 22
555, 833, 1250, 625, 938, 469, 704, 352, 176, 88, 44, 22
559, 839, 1259, 1889, 2834, 1417, 2126
567, 851, 1277, 1916, 958
571, 857, 1286
579, 869, 1304, 652, 326
583, 875, 1313, 1970, 985, 1478
591, 887, 1331, 1997, 2996, 1498, 749, 1124, 562, 281, 422
595, 893, 1340, 670
603, 905, 1358
607, 911, 1367, 2051, 3077, 4616, 2308, 1154, 577, 866, 433, 650, 325, 488, 244, 122, 61, 92, 46
615, 923, 1385, 2078
619, 929, 1394, 697, 1046
627, 941, 1412, 706, 353, 530, 265, 398
631, 947, 1421, 2132, 1066, 533, 800, 400, 200, 100, 50, 25, 38
639, 959, 1439, 2159, 3239, 4859, 7289, 10934
643, 965, 1448, 724, 362, 181, 272, 136, 68, 34, 17, 26, 13, 20, 10, 5, 8, 4, 2, 1
651, 977, 1466, 733, 1100, 550
655, 983, 1475, 2213, 3320, 1660, 830
663, 995, 1493, 2240, 1120, 560, 280, 140, 70
667, 1001, 1502
675, 1013, 1520, 760, 380, 190
679, 1019, 1529, 2294
687, 1031, 1547, 2321, 3482, 1741, 2612, 1306, 653, 980, 490, 245, 368, 184, 92, 46
691, 1037, 1556, 778, 389, 584, 292, 146, 73, 110
699, 1049, 1574
703, 1055, 1583, 2375, 3563, 5345, 8018, 4009, 6014
711, 1067, 1601, 2402, 1201, 1802, 901, 1352, 676, 338, 169, 254
715, 1073, 1610, 805, 1208, 604, 302
723, 1085, 1628, 814
727, 1091, 1637, 2456, 1228, 614
735, 1103, 1655, 2483, 3725, 5588, 2794, 1397, 2096, 1048, 524, 262
739, 1109, 1664, 832, 416, 208, 104, 52, 26, 13, 20, 10, 5, 8, 4, 2, 1
747, 1121, 1682, 841, 1262
751, 1127, 1691, 2537, 3806
759, 1139, 1709, 2564, 1282, 641, 962, 481, 722, 361, 542
763, 1145, 1718
771, 1157, 1736, 868, 434, 217, 326
775, 1163, 1745, 2618, 1309, 1964, 982
783, 1175, 1763, 2645, 3968, 1984, 992, 496, 248, 124, 62
787, 1181, 1772, 886
795, 1193, 1790
799, 1199, 1799, 2699, 4049, 6074, 3037, 4556, 2278
807, 1211, 1817, 2726
811, 1217, 1826, 913, 1370, 685, 1028, 514, 257, 386, 193, 290, 145, 218, 109, 164, 82, 41, 62
819, 1229, 1844, 922, 461, 692, 346, 173, 260, 130, 65, 98, 49, 74, 37, 56, 28, 14
823, 1235, 1853, 2780, 1390
831, 1247, 1871, 2807, 4211, 6317, 9476, 4738, 2369, 3554, 1777, 2666, 1333, 2000, 1000, 500, 250, 125, 188, 94
835, 1253, 1880, 940, 470
843, 1265, 1898, 949, 1424, 712, 356, 178, 89, 134
847, 1271, 1907, 2861, 4292, 2146, 1073, 1610, 805, 1208, 604, 302
855, 1283, 1925, 2888, 1444, 722, 361, 542
859, 1289, 1934
867, 1301, 1952, 976, 488, 244, 122, 61, 92, 46
871, 1307, 1961, 2942
879, 1319, 1979, 2969, 4454
883, 1325, 1988, 994, 497, 746, 373, 560, 280, 140, 70
891, 1337, 2006
895, 1343, 2015, 3023, 4535, 6803, 10205, 15308, 7654
903, 1355, 2033, 3050, 1525, 2288, 1144, 572, 286
907, 1361, 2042, 1021, 1532, 766
915, 1373, 2060, 1030
919, 1379, 2069, 3104, 1552, 776, 388, 194, 97, 146, 73, 110
927, 1391, 2087, 3131, 4697, 7046
931, 1397, 2096, 1048, 524, 26
939, 1409, 2114, 1057, 1586, 793, 1190
943, 1415, 2123, 3185, 4778, 2389, 3584, 1792, 896, 448, 224, 112, 56, 28, 14
951, 1427, 2141, 3212, 1606
955, 1433, 2150
963, 1445, 2168, 1084, 542
967, 1451, 2177, 3266, 1633, 2450, 1225, 1838
975, 1463, 2195, 3293, 4940, 2470
979, 1469, 2204, 1102
987, 1481, 2222
991, 1487, 2231, 3347, 5021, 7532, 3766
999, 1499, 2249, 3374
1003, 1505, 2258, 1129, 1694
1011, 1517, 2276, 1138, 569, 854
1015, 1523, 2285, 3428, 1714, 857, 1286
1023, 1535, 2303, 3455, 5183, 7775, 11663, 17495, 26243, 39365, 59048, 29524, 14762, 7381, 11072, 5536, 2768, 1384, 692, 346, 173, 260, 130, 65, 98, 49, 74, 37, 56, 28, 14
1027, 1541, 2312, 1156, 578, 289, 434, 217, 326

and so forth.

7.3 Residue classes for $C^h(s)$

$h = 2$
if $s \equiv$ 9 $(mod\ 48)$

$h = 3$
if $s \equiv$ 93 $(mod\ 96)$

$h = 4$
if $s \equiv$ 33, 165 $(mod\ 192)$

$h = 5$
if $s \equiv$ 81, 117, 237 $(mod\ 384)$

$h = 6$
if $s \equiv$ 129, 333, 405, 561, 645 $(mod\ 768)$

$h = 7$
if $s \equiv$ 429, 657, 837, 981, 1293, 1461, 1473, 1521 $(mod\ 1536)$

$h = 8$
if $s \equiv$ 177, 309, 513, 597, 1089, 1221, 1557, 1581, 1677, 1809, 1905, 2565, 3021 $(mod\ 3072)$

$h = 9$
if $s \equiv$ 813, 1041, 1077, 1281, 1749, 1797, 1989, 2241, 2253, 2445, 2481, 2673, 3393, 3765, 4677, 5133, 5361, 5397, 5805, 5973, 6033 $(mod\ 6144)$

$h = 105$
if $s \equiv$ 261, 717, 789, 1197, 2049, 2133, 2229, 2613, 2817, 3117, 3141, 3441, 3525, 3597, 3777, 3981, 4785, 5421, 5697, 6165, 7089, 7569, 7989, 8001, 8901, 9357, 9429, 9489, 9969, 10245, 10353, 10581, 10701, 11793 $(mod\ 12288)$

$h = 11$
if $s \equiv$ 273, 525, 1485, 2289, 2325, 2349, 4113, 4149, 5061, 5205, 5313, 5325, 6285, 6837, 6849, 6897, 7053, 7509, 7749, 7857, 8721, 8769, 9933, 11445, 11697, 12357, 12501, 12609, 12657, 13317, 13425, 15021, 15189, 15381, 16149, 16305, 16557, 16641, 17157, 17205, 17217, 17805, 17973, 18117, 18177, 18477, 18885, 20493, 21393, 21765, 22317, 22929, 23253, 23553, 24177 $(mod\ 24576)$

$h = 12$
if $s \equiv$ 945, 2757, 3345, 4557, 5301, 6405, 6417, 6513, 7629, 8193, 8277, 8433, 8877, 10125, 10161, 10257, 10497, 11469, 11841, 11973, 13893, 14913, 15501, 15573, 15621, 16113, 17937, 18033, 18753, 19149, 19221, 19569, 21045, 21333, 21549, 21573, 23493, 24597, 24621, 24717, 25029, 25329, 25941, 26001, 26421, 26433, 26637, 27153, 28365, 28461, 29073, 30789, 33813, 34317, 34497, 35505, 35637, 35649, 35925, 36141, 36237, 37557, 37569, 37773, 38577, 39825, 40965, 41073, 41301, 41685, 41985, 42417, 43221, 43533, 44037, 44205, 45237, 45333, 45357, 45825, 46449, 46533, 46869, 47157, 47277, 47877, 48321, 48693, 48897 $(mod\ 49152)$

$h = 135$
if $s \equiv$ 693, 705, 1029, 3093, 3285, 3333, 4269, 5517, 5553, 5685, 5745, 5889, 6357, 6933, 7173, 8373, 9105, 9261, 9585, 10005, 10413, 11265, 12033, 12813, 13041, 13233, 13713, 16401, 16437, 17601, 17613, 20037, 20481, 21525, 22209, 22545, 23733, 23757, 23949, 24897, 24945, 26181, 27309, 27477, 27789, 28593, 28677, 28785, 28929, 30225, 31041, 31173, 31245, 31437, 31857, 31917, 33537, 33861, 34605, 34869, 38709, 38721, 40749, 41925, 42069, 42165, 43149, 43377, 44721, 45585, 46797, 46989, 47025, 48429, 49221, 49365, 50061, 50865, 53013, 53589, 54021, 54069, 54081, 57909, 60117, 60165, 60357, 60981, 61581, 61713, 62805, 62925, 63729, 63765, 63789, 64017, 64197, 65229, 67653, 67845, 67857, 69069, 69873, 70209, 72369, 72501, 72513, 74865, 76353, 76689, 77553, 78549, 78849, 79557, 79917, 81933, 82197, 82221, 82773, 83205, 83397, 84369, 85185, 85617, 86037, 86061, 86469, 88077, 89361, 90513, 90573, 92853, 92865, 92913, 93069, 93525, 94293, 94893, 95757, 96513, 97365, 97713, 97857, 97989 $(mod\ 98304)$

$h = 14$
if $s \equiv$ 2061, 3789, 3825, 4209, 4497, 6213, 6597, 6669, 9477, 10929, 11061, 11073, 11349, 11889, 12741, 14349, 15249, 16785, 17409, 18645, 18801, 19185, 19797, 20145, 20661, 20757, 20781, 25521, 27921, 28437, 32769, 32853, 32949, 33333, 33681, 34581, 35781, 35841, 36045, 37137, 37809, 38229, 38469, 40077, 40149, 40197, 40653, 41013, 42513, 42609, 43077, 43329, 47925, 47937, 48309, 48333, 49905, 50289, 50577, 50757, 52365, 53037, 53169, 54801, 55617, 56433, 57357, 57621, 57645, 59793, 60609, 62349, 65325, 65997, 67725, 69717, 70161, 70317, 71733, 71937, 72453, 73281, 76485, 77013, 77061, 78165, 79029, 79413, 79473, 81357, 82161, 82989, 83853, 83889, 84693, 84741, 85557, 86769, 88641, 88773, 91341, 93645, 93765, 94449, 95937, 98325, 98349, 98445, 98625, 98757, 99669, 100881, 100929, 101205, 102405, 104133, 105645, 107265, 107349, 108045, 108333, 108597, 110613, 110637, 111285, 111297, 115413, 115797, 115893, 116145, 117765, 120177, 120261, 120333, 120597, 120717, 120753, 121005, 121941, 122049, 122625, 123093, 123573, 123585, 125973, 127437, 127749, 128397, 128433, 129237, 130053, 131757, 132465, 133137, 133293, 133377, 134721, 134853, 134913, 135309, 135693, 136653, 137457, 137745, 140481, 142029, 142101, 142449, 143361, 143937, 144429, 144453, 145425, 146097, 147825, 149301, 149313, 150189, 150417, 151809, 152277, 152577, 153645, 154317, 156693, 156741, 157125, 157377, 159021, 159117, 161457, 161589, 161601, 163089, 163845, 163953, 164181, 164805, 166413, 166581, 166593, 166797, 167085, 168705, 169677, 171309, 171441, 171573, 171717, 172101, 172941, 173061, 173745, 176301, 176949, 176961, 177921, 178437, 178449, 178545, 178965, 185745, 186645, 186669, 188145, 188433, 190725, 190737, 193077, 193365, 193557, 195525, 195981 $(mod\ 196608)$

$h = 15$
if $s \equiv$ 8469, 8493, 9237, 9285, 10641, 12429, 13197, 13773, 14133, 14145, 17349, 18693, 18705, 18957, 22221, 23853, 24129, 25485, 27333, 27861, 27909, 28401, 32205, 33837, 36465, 37317, 37617, 40209, 41361, 43269, 43281, 44613, 46785, 48069, 49473, 52053, 52941, 52977, 53253, 56085, 56493, 57525, 58113, 59181, 61041, 62385, 62865, 65553, 65589, 65937, 66645, 66741, 68949, 70677, 71025, 71445, 71565, 71601, 75333, 76941, 77073, 78597, 80193, 80325, 80397, 81009, 82005, 82101, 82833, 83733, 83985, 84993, 86157, 86961, 88305, 89901, 90165, 91761, 94737, 94785, 95301, 96945, 99729, 99909, 100149, 100161, 101265, 101517, 103425, 104493, 104769, 109269, 109317, 109509, 109869, 110133, 110865, 111957, 114381, 114477, 115029, 117429, 117441, 119025, 119313, 120885, 122289, 123909, 124017, 125505, 127149, 127317, 128709, 128769, 129285, 129297, 129813, 133521, 133845, 133893, 135189, 135213, 138993, 143445, 143601, 144213, 144405, 144909, 147009, 148149, 148161, 150033, 150081, 151953, 152973, 153009, 153141, 153201, 153285, 153813, 154053, 154125, 154629, 156501, 156717, 157869, 158805, 159489, 159765, 159789, 164565, 167493, 167601, 168117, 169413, 169485, 169665, 171093, 172353, 172401, 172725, 172737, 172977, 176133, 177549, 177585, 178893, 179205, 179373, 180993, 182061, 182325, 182445, 183237, 183873, 184005, 184065, 185925, 187533, 188109, 189633, 190785, 192513, 193581, 196677, 196821, 196977, 197517, 197745, 198321, 199341, 200493, 200961, 201477, 201525, 201537, 203469, 204813, 206529, 208065, 208269, 211185, 211221, 211245, 211473, 212997, 213105, 213453, 216237, 217173, 217665, 217773, 217857, 219393, 219825, 219909, 221253, 222897, 224145, 226005, 226101, 226113, 226305, 226869, 226929, 227373, 227697, 229617, 230661, 230853, 235533, 235797, 235821, 236097,

238797, 240309, 240321, 240369, 240525, 240981, 242229, 245169, 245445, 245781, 245805, 246213, 246789, 250029, 251649, 251973, 254865, 255237, 255501, 255765, 256053, 256689, 256821, 256833, 257025, 258501, 258741, 258753, 260109, 261009, 263169, 263601, 264945, 265221, 266517, 266541, 268461, 269493, 269517, 269709, 270081, 270549, 274353, 274545, 275985, 284109, 285201, 285909, 285957, 288909, 289137, 289485, 289905, 290481, 294069, 294093, 295665, 298773, 298929, 299349, 299733, 300561, 302193, 303669, 304581, 308913, 309957, 310545, 313413, 313485, 314253, 314829, 318261, 318273, 319029, 319173, 324789, 327693, 327957, 327981, 328533, 329613, 329649, 330945, 331317, 333201, 334533, 335889, 336333, 339405, 340653, 342273, 343125, 343437, 344205, 345429, 346125, 347157, 347349, 347397, 348273, 353649, 356877, 357105, 361665, 361677, 362709, 362865, 364545, 366609, 367809, 371373, 371541, 371733, 372501, 372993, 373197, 374997, 376833, 377397, 377517, 377925, 378225, 379137, 380109, 381201, 381453, 382293, 382773, 382785, 385989, 386133, 386229, 386577, 387141, 387861, 390861, 391053, 391089, 391185, 391989, 392001, 392493 $(mod\ 393216)$

$h = 16$
if $s \equiv$ 1845, 1857, 2961, 5121, 5649, 6189, 8757, 11565, 16725, 17841, 18501, 18573, 19125, 19137, 24321, 24789, 25605, 27669, 28845, 30981, 30993, 34161, 34701, 34737, 35073, 37389, 38349, 40689, 43725, 43797, 44145, 44493, 45909, 46101, 47121, 47361, 50517, 53649, 53973, 54669, 54705, 55509, 55749, 55821, 61185, 63153, 63285, 63297, 66501, 66765, 69813, 72897, 73005, 73269, 73413, 74097, 74637, 76629, 76821, 78285, 80085, 80589, 82605, 83313, 84933, 86541, 89229, 90129, 92421, 92433, 92481, 92949, 96141, 96177, 96273, 100017, 102093, 102129, 102189, 102513, 103221, 103233, 106509, 109761, 112077, 112437, 112449, 112917, 112941, 115089, 115149, 115653, 119469, 121605, 122157, 131073, 131157, 131253, 131313, 131637, 132357, 132885, 134349, 135441, 136533, 137793, 140817, 140913, 141381, 141573, 141585, 142065, 142677, 147477, 147501, 150669, 151245, 151281, 153921, 154389, 157197, 158097, 160437, 160449, 160653, 163629, 163857, 164241, 166917, 167253, 170037, 170157, 171189, 171213, 172245, 175377, 177681, 179661, 180309, 180405, 181137, 182997, 183045, 183297, 190065, 190605, 191601, 192753, 196929, 198033, 199185, 199233, 199509, 199821, 201045, 201429, 202437, 203073, 205569, 206637, 208917, 208941, 209169, 212241, 212685, 212781, 216525, 218481, 218565, 218637, 218901, 219021, 219057, 219189, 220245, 220725, 220869, 221877, 221889, 222321, 225621, 226053, 228357, 230229, 231441, 231597, 231825, 232149, 232197, 234897, 235761, 241749, 241905, 242241, 244401, 247605, 247617, 247821, 248337, 248385, 248721, 248853, 249969, 250113, 250881, 251445, 251505, 251589, 251949, 254805, 255345, 257325, 257421, 258069, 258093, 258573, 264885, 264897, 267009, 267717, 267789, 268305, 270405, 270657, 271029, 271041, 271365, 274605, 274689, 276741, 276753, 276849, 277269, 277509, 278529, 279093, 279297, 280365, 280629, 280749, 282177, 284469, 284481, 286449, 287685, 287937, 288837, 290817, 291885, 294189, 297645, 299409, 299781, 304833, 305841, 305973, 305985, 306261, 306573, 307653, 308109, 309261, 309489, 309777, 310161, 311301, 312321, 313605, 313617, 314541, 315573, 315669, 315693, 315969, 318129, 319041, 319557, 322245, 322449, 323313, 324609, 325677, 326001, 328749, 329157, 334989, 335061, 335109, 335565, 336273, 338241, 338613, 338625, 338829, 339525, 341697, 344817, 345093, 345201, 346965, 347949, 348081, 348165, 348333, 351405, 352269, 352437, 355125, 355137, 355521, 358413, 360909, 364629, 364821, 364845, 365229, 366357, 366477, 366513, 367365, 368013, 371853, 373941, 374325, 374385, 375105, 375921, 377073, 383553, 383685, 384213, 384261, 384813, 387441, 390213, 393237, 393261, 393357, 393669, 393969, 394581, 397077, 397233, 400497, 402957, 403509, 404181, 404229, 405045, 406197, 406209, 412677, 413937, 415917, 416565, 416577, 416961, 417201, 420417, 422349, 423093, 423621, 423681, 424149, 424725, 425997, 426261, 426285, 426669, 429249, 429621, 430101, 430125, 432657, 432837, 434637, 438957, 439821, 442509, 443061, 443073, 445653, 445701, 449541, 451605, 451653, 452037, 452781, 453717, 455409, 458001, 461325, 461709, 462513, 464589, 467889, 477441, 480657, 480837, 483021, 484437, 484533, 490437, 490893, 491589, 492429, 492657, 493581, 500757, 500805, 503409, 503949, 510165, 510321, 510477, 511317, 512085, 513741, 514305, 517005, 519441, 519957, 521781, 521841, 525201, 527361, 529329, 530445, 531729, 532533, 533709, 539445, 539457, 539589, 541125, 542097, 542277, 549141, 549165, 549777, 550677, 550965, 551937, 552561, 554385, 558513, 561681, 562197, 564801, 564993, 568005, 568533, 568581, 569265, 569685, 571845, 571917, 574509, 575253, 578289, 578481, 579021, 580113, 581685, 584397, 585285, 587457, 591429, 593925, 597165, 598869, 599493, 601029, 603477, 603825, 604869, 606933, 607317, 607413, 609165, 610833, 619917, 619953, 626241, 626373, 626433, 626829, 630801, 632001, 634881, 635949, 635973, 638037, 638349, 638529, 639345, 641709, 645837, 648237, 648897, 655365, 655473, 655701, 656085, 656577, 657621, 657777, 658605, 659013, 659457, 661185, 662613, 662961, 665265, 666285, 667413, 667653, 668469, 668481, 669069, 669105, 669441, 670893, 672837, 675021, 675525, 675585, 676113, 677205, 678165, 678189, 681489, 684597, 684885, 685077, 685773, 686901, 686913, 688341, 688497, 692481, 694341, 694725, 694797, 694989, 696597, 696621, 697605, 698769, 704625, 707313, 708273, 709377, 713649, 714417, 715989, 716037, 717525, 717621, 717633, 720333, 723909, 724593, 725445, 725745, 726597, 727317, 727341, 733749, 736437, 736461, 736689, 736965, 737601, 742929, 743169, 743493, 744561, 746241, 746757, 747309, 748209, 750021, 750513, 752529, 753717, 754689, 754773, 754869, 755853, 756465, 759153, 760065, 763461, 766065, 766725, 771981, 772017, 772113, 773685, 774285, 776433, 779469, 781773, 782001, 782865, 782913, 785073, 785589, 785613 $(mod\ 786432)$

and so forth.

7.4 Residue classes for $C^t(s)$

$t = 2$
if $s \equiv$ 27, 91 $(mod\ 96)$

$t = 3$
if $s \equiv$ 19, 39, 147, 103 $(mod\ 192)$

$t = 4$
if $s \equiv$ 55, 67, 111, 183, 195, 235, 363, 367 $(mod\ 384)$

$t = 5$
if $s \equiv$ 139, 159, 163, 207, 243, 327, 415, 463, 471, 499, 583, 651, 675, 727 $(mod\ 768)$

$t = 6$
if $s \equiv$ 51, 99, 259, 279, 427, 447, 559, 655, 715, 771, 775, 939, 991, 1015, 1071, 1075, 1123, 1167, 1227, 1287, 1303, 1471, 1503, 1527 $(mod\ 1536)$

$t = 7$
if $s \equiv$ 31, 135, 175, 291, 319, 331, 375, 627, 639, 855, 967, 1119, 1159, 1203, 1315, 1323, 1359, 1399, 1431, 1551, 1651, 1663, 1803, 1879, 1923, 2019, 2079, 2143, 2223, 2227, 2347, 2367, 2379, 2383, 2455, 2575, 2827, 2947, 3015, 3043 $(mod\ 3072)$

$t = 8$
if $s \equiv$ 43, 63, 199, 223, 271, 351, 355, 435, 519, 523, 663, 703, 799, 895, 903, 1027, 1327, 1455, 1483, 1615, 1707, 1783, 1791, 1935, 2091, 2167, 2179, 2247, 2271, 2319, 2403, 2419, 2571, 2635, 2647, 2751, 2847, 2943, 3075, 3375, 3379, 3531, 3607, 3619, 3663, 3811, 3831, 4159, 4215, 4227, 4447, 4467, 4531, 4615, 4683, 4695, 4759, 4999, 5427,

5551, 5655, 5667, 5803, 5859, 5887, 6031 $(mod\ 6144)$

$t = 9$
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256303, 256543, 256867, 256911, 257047, 257631, 257919, 258315, 258639, 258859, 259359, 259735, 259935, 260019, 260119, 260983, 261367, 261903, 262003, 262063, 263175, 264931, 264975, 265227, 265239, 265903, 266275, 266623, 266755, 267307, 267775, 267927, 268287, 268375, 268719, 269431, 269443, 269559, 269827, 271255, 271395, 271435, 271447, 271903, 272691, 273271, 273423, 274311, 274351, 274431, 274687, 274863, 274915, 274947, 275191, 275263, 275491, 275575, 275911, 276271, 276567, 277015, 277167, 277635, 278079, 278167, 278719, 279159, 279175, 279895, 280287, 280767, 280963, 280975, 281335, 281419, 281871, 282055, 282111, 282411, 282415, 282495, 282751, 283999, 284019, 284083, 284203, 284535, 284751, 284899, 285615, 285739, 286723, 287407, 287871, 287875, 288343, 288883, 288919, 289023, 289455, 289963, 290019, 290503, 290679, 290851, 291423, 291759, 291903, 292087, 292111, 292195, 292651, 293911, 295395, 295423, 295447, 295519, 295651, 295743, 295935, 295999, 296287, 296479, 296583, 296967, 297591, 297607, 298207, 298231, 298627, 298647, 298675, 298767, 298851, 299007, 299295, 300303, 300835, 300843, 301791, 303411, 303447, 303619, 304051, 304075, 304135, 304527, 304611, 305031, 305047, 305155, 305239, 306291, 306763, 306775, 307215, 307315, 307327, 307371, 307711, 307911, 308355, 308395, 308659, 308871, 308935, 309055, 309555, 310015, 310791, 311175, 311751, 311871, 312111, 313119, 313239, 313419, 313495, 313687, 314383, 314739, 314755, 314767, 315403, 315823, 316423, 316935, 316959, 317119, 317215, 317311, 317995, 318015, 318543, 318579, 318615, 319659, 319971, 320199, 320883, 321331, 321879, 322831, 322915, 322947, 322951, 322959, 323071, 323203, 323223, 323247, 323371, 323479, 323967, 324531, 325503, 325711, 326091, 326179, 326187, 326671, 327343, 327687, 327703, 327807, 327907, 328063, 328111, 328323, 328591, 328975, 329311, 329395, 329515, 329551, 330031, 330271, 331455, 332367, 332535, 332559, 332659, 332835, 332931, 333087, 333187, 333255, 333615, 333747, 333831, 334735, 335455, 335895, 336427, 337411, 337419, 337483, 337935, 337999, 338511, 339007, 339031, 339231, 339327, 340815, 341727, 342207, 342283, 342883, 343903, 343959, 344079, 344215, 344407, 345487, 345603, 346135, 346879, 347223, 347491, 347767, 349107, 349263, 350335, 350551, 350899, 350991, 351055, 351631, 352051, 352075, 353103, 353667, 353671, 353923, 353971, 355407, 355551, 355843, 356811, 356899, 356907, 357091, 357451, 357463, 357895, 358647, 358975, 359091, 359647, 359691, 360223, 360703, 360715, 361131, 361135, 361215, 361311, 361671, 361983, 362059, 362335, 362955, 363363, 363379, 363583, 363903, 365151, 365463, 365631, 365823, 366399, 366615, 367279, 368139, 368535, 369783, 370183, 370495, 371071, 371263, 371371, 371427, 371491, 371911, 371979, 372579, 373191, 373419, 373551, 373555, 373599, 373759, 373959, 374283, 374647, 374731, 375171, 375175, 375519, 375727, 376243, 376291, 376323, 376627, 376831, 376867, 377931, 377943, 378243, 378247, 378411, 379471, 380191, 381475, 381483, 381663, 381747, 381759, 382795, 383367, 384435, 384799, 384883, 385023, 385215, 385279, 385935, 386047, 386059, 386655, 386695, 386847, 386895, 387147, 387375, 387615, 387763, 387939, 387955, 388119, 389931, 390271, 390655, 390807, 390991, 391191, 391471, 391711, 391759, 392055, 392439, 392479, 393075, 393079, 393135 $(mod\ 393216)$

and so forth.

7.5 Stopping time residue classes

$\sigma(s) = 1$
if $s \equiv 0 \pmod 2$

$\sigma(s) = 2$
if $s \equiv 1 \pmod 4$

$\sigma(s) = 4$
if $s \equiv 3 \pmod{16}$

$\sigma(s) = 5$
if $s \equiv 11, 23 \pmod{32}$

$\sigma(s) = 7$
if $s \equiv 7, 15, 59 \pmod{128}$

$\sigma(s) = 8$
if $s \equiv 39, 79, 95, 123, 175, 199, 219 \pmod{256}$

$\sigma(s) = 10$
if $s \equiv 287, 347, 367, 423, 507, 575, 583, 735, 815, 923, 975, 999 \pmod{1024}$

$\sigma(s) = 12$
if $s \equiv$ 231, 383, 463, 615, 879, 935, 1019, 1087, 1231, 1435, 1647, 1703, 1787, 1823, 1855, 2031, 2203, 2239, 2351, 2587, 2591, 2907, 2975, 3119, 3143, 3295, 3559, 3675, 3911, 4063 $(mod\ 4096)$

$\sigma(s) = 13$
if $s \equiv$ 191, 207, 255, 303, 539, 543, 623, 679, 719, 799, 1071, 1135, 1191, 1215, 1247, 1327, 1563, 1567, 1727, 1983, 2015, 2075, 2079, 2095, 2271, 2331, 2431, 2607, 2663, 3039, 3067, 3135, 3455, 3483, 3551, 3687, 3835, 3903, 3967, 4079, 4091, 4159, 4199, 4223, 4251, 4455, 4507, 4859, 4927, 4955, 5023, 5103, 5191, 5275, 5371, 5439, 5607, 5615, 5723, 5787, 5871, 5959, 5979, 6047, 6215, 6375, 6559, 6607, 6631, 6747, 6815, 6983, 7023, 7079, 7259, 7375, 7399, 7495, 7631, 7791, 7847, 7911, 7967, 8047, 8103 $(mod\ 8192)$

$\sigma(s) = 15$
if $s \equiv$ 127, 411, 415, 831, 839, 1095, 1151, 1275, 1775, 1903, 2119, 2279, 2299, 2303, 2719, 2727, 2767, 2799, 2847, 2983, 3163, 3303, 3611, 3743, 4007, 4031, 4187, 4287, 4655, 5231, 5311, 5599, 5631, 6175, 6255, 6503, 6759, 6783, 6907, 7163, 7199, 7487, 7783, 8063, 8187, 8347, 8431, 8795, 9051, 9087, 9371, 9375, 9679, 9711, 9959, 10055, 10075, 10655, 10735, 10863, 11079, 11119, 11567, 11679, 11807, 11943, 11967, 12063, 12143, 12511, 12543, 12571, 12827, 12967, 13007, 13087, 13567, 13695, 13851, 14031, 14271, 14399, 14439, 14895, 15295, 15343, 15839, 15919, 16027, 16123, 16287, 16743, 16863, 16871, 17147, 17727, 17735, 17767, 18011, 18639, 18751, 18895, 19035, 19199, 19623, 19919, 20079, 20199, 20507, 20527, 20783, 20927, 21023, 21103, 21223, 21471, 21727, 21807, 22047, 22207, 22655, 22751, 22811, 22911, 22939, 23231, 23359, 23399, 23615, 23803, 23835, 23935, 24303, 24559, 24639, 24647, 24679, 25247, 25503, 25583, 25691, 25703, 25831, 26087, 26267, 26527, 26535, 27111, 27291, 27759, 27839, 27855, 27975, 28703, 28879, 28999, 29467, 29743, 29863, 30311, 30591, 30687, 30715, 30747, 30767, 30887, 31711, 31771, 31899, 32155, 32239, 32575, 32603 $(mod\ 32768)$

$\sigma(s) = 16$
if $s \equiv$ 359, 479, 559, 603, 767, 859, 1179, 1183, 1351, 1519, 1535, 1627, 2367, 2407, 2495, 2671, 2687, 2791, 2887, 2927, 3103, 3239, 3487, 3535, 3695, 3815, 4319, 4335, 4379, 4635, 4775, 4799, 4815, 4895, 4991, 5087, 5343, 5375, 5423, 5583, 5663, 5823, 5863, 6207, 6247, 6555, 6639, 6703, 6975, 7015, 7103, 7231, 7451, 7471, 7551, 7711, 7835, 7871, 7931, 8095, 8263, 8551, 8671, 8863, 9119, 9199, 9319, 9543, 9599, 9819, 9935, 10151, 10559, 10727, 10907, 11035, 11247, 11431, 11727, 11823, 11887, 12007, 12319, 12495, 12615, 12775, 12799, 13279, 13339, 13535, 13615, 13671, 13855, 13927, 13951, 14015, 14207, 14303, 14363, 14383, 14503, 14543, 14747, 15103, 15167, 15207, 15423,

15487, 15515, 15599, 15643, 15743, 15771, 15855, 16191, 16411, 16431, 16455, 16511, 16635, 16831, 17055, 17127, 17135, 17223, 17311, 17391, 17479, 17511, 17659, 18159, 18343, 18523, 18559, 18919, 19099, 19111, 19135, 19151, 19231, 19367, 19547, 19687, 19707, 20127, 20207, 20511, 20591, 20687, 20807, 21039, 21595, 21615, 21695, 21735, 22015, 22119, 22399, 22495, 22555, 22575, 22695, 22887, 23143, 23167, 23583, 23663, 23707, 23711, 23743, 23963, 24047, 24383, 24571, 24703, 24731, 24815, 25371, 25415, 25471, 25599, 25671, 25851, 26015, 26063, 26343, 26351, 26367, 26439, 26459, 26619, 27039, 27119, 27303, 27343, 27423, 27559, 27675, 27739, 27879, 27903, 27951, 28095, 28191, 28319, 28327, 28351, 28447, 28507, 28527, 28927, 29087, 29231, 29631, 29807, 29823, 29887, 30079, 30207, 30235, 30415, 30575, 30655, 30971, 30975, 31079, 31199, 31335, 31359, 31471, 31727, 31775, 32223, 32283, 32303, 32703, 32763, 32859, 32923, 33007, 33087, 33255, 33531, 33663, 34111, 34151, 34255, 34271, 34535, 34631, 34651, 34927, 35023, 35231, 35279, 35311, 35419, 35579, 35583, 36143, 36159, 36383, 36519, 36543, 36635, 36639, 36719, 36891, 36911, 37119, 37167, 37311, 37407, 37467, 37487, 37607, 37735, 38047, 38171, 38271, 38427, 38607, 38847, 39039, 39135, 39195, 39295, 39535, 39615, 39919, 40039, 40187, 40351, 40415, 40495, 40687, 40943, 41023, 41063, 41183, 41243, 41447, 41627, 41723, 42075, 42215, 42239, 42303, 42343, 42471, 42651, 42911, 43071, 43111, 43215, 43335, 43471, 43611, 43775, 43967, 44143, 44223, 44239, 44359, 44699, 44959, 45083, 45103, 45223, 45359, 45503, 45535, 45599, 45679, 45799, 45851, 46127, 46247, 46407, 47099, 47231, 47327, 47387, 47423, 47487, 47807, 48095, 48155, 48295, 48379, 48879, 48987, 49135, 49215, 49255, 49311, 49563, 49567, 49983, 50143, 50267, 50303, 50407, 50663, 50843, 50847, 51055, 51103, 51271, 51431, 51451, 51455, 51611, 51871, 51951, 52031, 52071, 52335, 52415, 52431, 52507, 52551, 52735, 52763, 53159, 53183, 53319, 53339, 53439, 53887, 53919, 54043, 54303, 54319, 54375, 54439, 54751, 55207, 55291, 55327, 55407, 55535, 55963, 56059, 56191, 56287, 56315, 56347, 56639, 56935, 57179, 57215, 57375, 57671, 57755, 57759, 57839, 57947, 58175, 58203, 58495, 58523, 58527, 58863, 58983, 59247, 59263, 59463, 59559, 59623, 59643, 59647, 60015, 60063, 60143, 60231, 60271, 60571, 60831, 60911, 60955, 61135, 61351, 61375, 61531, 61631, 61663, 61723, 61979, 62119, 62159, 62239, 62279, 62719, 62943, 63023, 63335, 63519, 63551, 63591, 63599, 64047, 64167, 64207, 64251, 64287, 64447, 64507, 64831, 64871, 65127, 65179, 65183, 65275, 65407, 65439 $(mod\ 65536)$

and so forth.

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